07

Tying a Shoelace

Heuristics to highlight:

- Use suitable numbers (instead of algebra)
- Use equations/algebra
- Aim for subgoals.

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A man is in a hurry to get on a plane. While walking as fast as he can in the airport, he notices that the lace on one of his shoes is untied.

The untied shoelace will not slow him down but he must arrive at the embarkation gate with the lace tied.

There are travelators (moving horizontal pedestrian carriers) in some sections of the airport along his way.

Should he tie his lace on 'stationary' ground or on the travelator?

Source: Dr Toh Tin Lam, Projects OER 32/08 TTL and OER 22/12 TTL funded by the Education Research Funding Programme, National Institute of Education (NIE), Nanyang Technological University, Singapore. Adapted by Knowledge Mobilisation Unit, Office of Education Research, NIE, 2017.

SUITABLE HINTS FOR POLYA STAGES I, II AND IV

I) UNDERSTAND THE PROBLEM

(c) Write down the heuristics you used to understand the problem.
Act it out – Observe that he can walk on the travelator.

II) DEVISE A PLAN

(a) Write down the key concepts that might be involved in solving the problem. Speed.

(c) Write out each plan concisely and clearly.

Plan 1 *Use suitable numbers (instead of algebra) – let the speed of the man = speed of travelator = 1m/s; length of ground = length of travelator =10m; time taken to tie shoelace = 1s. Work out the time taken for the two cases.*

Plan 2 *Use equations/algebra – let the speed of the man = s1 m/s, speed of the travelator = s2 m/s, etc.*

IV) CHECK AND EXPAND

(a) Write down how you checked your solution.
By checking with another set of suitable numbers.
(b) Write down a sketch of any alternative solution(s) that you can think of.
By considering the time 'saved' when tying the shoelace on the travelator
(c) Give at least one adaptation, extension or generalisation of the problem.

Adaptation:

Assuming that the walking speed of the man is 4m/s, the speed of the travelator is 2m/s, the time taken to tie his shoelace is 5s, and that he can completely tie his shoelace if he starts doing so at the beginning of any stretch of travelator, how much faster would he arrive at the embarkation gate if he should he tie his lace on the travelator rather than on 'stationary' ground?

Generalisation:

Suppose we do not assume that he will complete tying his shoelace within the time spent on the travelator.

Extension:

A man is in a hurry to get on a plane. While walking as fast as he can in the airport, he notices that the lace on one of his shoes is untied.

The untied shoelace will not slow him down but he must arrive at the embarkation gate with the lace tied. There are travelators (moving horizontal pedestrian carriers) in some sections of the airport along his way and he must take a lift. Should he tie his lace on 'stationary' ground, on the travelator, or in the lift?

SOLUTIONS AND ASSESSMENT NOTES

SOLUTION #1

Let the walking speed of the man be s_1 m/s, the speed of the travelator be s_2 m/s, the time taken to tie his shoelace is t s, the distance of 'stationary' ground is d_1 m and the distance on the travelator be d_2 m. Suppose he ties on 'stationary' ground.

Time to tie lace = t.
Time to traverse d_1 m of 'stationary ground = d_1/s_1
Time to traverse d_2 m of travelator = $d_2/s_1 + s_2$
Thus, time taken to reach the gate = $d_1/s_1 + d_2/s_2$
Suppose he ties on the travelator.
Time to traverse d_1 m of 'stationary ground = d_1/s_1
Time to tie lace = t.
Distance travelled on travelator while tying lace = ts_2
Time to traverse remaining (d_2 - ts_2) m of travelator = d_2 - $ts_2/s_1 + s_2$
Thus, time taken to reach the gate = $t + d_1/s_1 + d_2$ - $st_2/s_1 + s_2 < t + d_1/s_1 + d_2/s_1 + s_2$

SOLUTION #2

No matter where he ties his shoelace, he has to spend the same amount of time tying his shoelace and traversing the 'stationary' ground. If he ties on the travelator, he 'saves' the distance that he has moved on the travelator while tying. Thus because of there being less distance to traverse, the time spent traversing the travelator if he ties on the travelator is less than if he ties on 'stationary' ground.

POSSIBLE STUDENT RESPONSES

Students use their intuition to insist on any one of 3 possibilities, i.e. better on ground, better on travelator, no difference. All 3 responses have been observed in real life. Insist that student be rigorous in their proof.

Assessment notes

Only proofs that meet the rigour of the solutions 1 or 2 are acceptable.