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Helping Math Learners

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Big Idea

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Understanding the Math Learner

To improve Math education in Singapore, much research has been done to understand how teachers teach Math. But in recent years, researchers have homed in on the other part of the equation: the learners and how they learn.

A doyenne of Math education in Singapore, Professor Berinderjeet Kaur has been a witness to how teaching and learning in Math has evolved over more than 30 years.

Two decades ago, the focus of research was on the teachers: how they taught their students, and what the common errors and misconceptions of their students are. The key question was: "How can they teach differently, so that the students don't make these mistakes?"

Ten years later, there was a shift in thinking among researchers, Prof Kaur recalls. A new paradigm began to take shape—with the learners as the focus.

A Holistic View of Math Education

During that time, several international studies looked at student achievement and performance. "There was a concerted push towards a holistic look at teaching and learning," Prof Kaur says. "We started looking at the interaction between three factors: the knowledge of Math, teacher and student—they form a triangle."

Prof Kaur and her colleagues from the Mathematics and Mathematics Education Academic Group (MME AG) in NIE were part of that effort. In 2004, they joined an international research collaboration called the Learner's Perspective Study, led by Professor David Clarke of the University of Melbourne in Australia.

"We learned a lot because that study positioned the learner very significantly in the interactive patterns," Prof Kaur notes. The researchers video-recorded Math lessons as they unfolded in the Singapore classrooms.

> How much we know is proportionate to how much research we've done, especially in the local context.
>
> - Prof Berinderjeet Kaur, *Mathematics and Mathematics Education Academic Group*

NIE National Institute of Education Singapore — An Institute of Nanyang Technological University

In this issue

RESEARCH within REACH

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Prof Berinderjeet Kaur (left) and Assoc Prof Toh Tin Lam believe that innovative approaches are needed to educate the new generation of students.

"We collected data from the students, and asked them questions about what they were doing during the lesson. That informed us a lot on our understanding of the learner in the classroom."

Looking beyond Grades

Today, the psyche of our students has become a matter of interest to educators and researchers.

Prof Kaur, for one, often reminds her student teachers during their pre-service education that they have to know the child first before teaching them. "As I always tell them, you must touch the students' hearts first, and then the mind. Don't go charging straight for the mind! If you haven't touched their hearts, you haven't won them over."

Deputy Head of MME Associate Professor Toh Tin Lam notices that school leaders and teachers today are concerned about more than just Math scores.

"They're not looking at performance in exams as the only indicator. They're also looking at other aspects of the students, such as self-motivation or academic self-concept."

Tin Lam has just started a project with researchers from other disciplines, such as psychology and social work, to look at alternative teaching in Math and how they might affect students' mathematical self-concept, motivation and achievement (See "Motivating Students in Math" in the online version of this issue).

"These are some other indicators we're working on," he says. "I believe on the whole, schools have become more receptive to what we're doing." In fact, they are often keen to participate in research projects by NIE researchers.

"Somehow we've managed to convince them what we're doing is relevant, through pre- and in-service education," he adds.

> We should try to see things in a different light, especially when it comes to the less traditional groups of learners.
>
> - Assoc Prof Toh Tin Lam, *Mathematics and Mathematics Education Academic Group*

High Performers and Low Attainers

After a decade of studying Math learners, how much do we know of them?

"How much we know is proportionate to how much research we've done, especially in the local context," Prof Kaur comments. "We have a fair bit but still not sufficient."

This is especially so as Singapore students span a wide spectrum of abilities. It is well known that they are chart-toppers in large-scale international assessments. Singapore ranked second in Math literacy skills in Programme for International Student Assessment (PISA) 2012 and first in Math for Trends in International Mathematics and Science Study (TIMSS) 2011.

There are many who do well in Math. "But as a society, we should not just be proud of these; we have to take care of the others," Prof Kaur notes. She and some colleagues did a research project on low attainers in Math (See our previous article, "Helping Low Attainers to Keep Up" in *SingTeach*, Issue 31).

There were two findings that she felt were significant.

"They were keen to learn because in class, their teachers often made them feel they can do the Math." But come assessment time, they struggled. The tests were often challenging even for better students. Prof Kaur suggests that the harder questions should be posed to these students gradually, so that they can experience success first and build their confidence.

Something else she noted was a mismatch between how these students wanted to learn and how the teachers wanted to teach.

"They wanted to learn through a lot of play, with manipulatives and some guidance," says Prof Kaur. But the teachers tended to prefer more traditional modes of teaching and direct instruction.

MOE were receptive to their findings, and implemented interventions that encouraged teachers to show more empathy and give students more space to express themselves.

Future Research

Studying learners at one end of the spectrum provides only a partial picture. Prof Kaur, Tin Lam and their colleagues hope to fill the gaps by proposing a new research project on the curriculum initiatives implemented under the current Math syllabuses. As part of the research, they want to look at lower secondary students of all abilities.

The team will video-record both teachers in action and students in the classroom. After that, they will show the videos to the students and ask them: *Why was the teacher doing this? Does it help you? Which part of the lesson was important for you?*

"You may find that our learners are actually very interesting," Prof Kaur says. "They may be very quiet throughout the lesson, but if you give them the questions, they can do them!"

For Tin Lam, another way to help learners is to think of education in a different way—as a form of persuasion or communication.

Theories in other fields, such as communications, persuasion and advertising, could possibly provide another lens for looking at how Math is taught, in particular, to lower achieving students.

For example, if students are weak in algebra, a teacher can expose them to even more of it. Because of mere-exposure effect, which predicts that people will develop a preference for things or people that they are familiar with, students may grow to like it.

"But at the same time, make sure they don't develop negative feelings about it," says Tin Lam. "This is what advertising tries to do!"

These are just some initial possibilities that he is entertaining. In educating the new generation of students with different needs, perhaps such innovative ways of thinking are in order. As Tin Lam puts it, "We should try to see things in a different light, especially when it comes to the less traditional groups of learners."

Prof Berinderjeet Kaur is a Professor in the Mathematics and Mathematics Education Academic Group (MME AG) at NIE. Her primary interest is the teaching and learning (pedagogy) of Math. **Assoc Prof Toh Tin Lam** is Deputy Head of MME AG and Associate Professor at NIE. His research interests include mathematical problem solving, use of alternative pedagogy in teaching Math and developing teachers' mathematical content knowledge.

Research

Improving Mathematical Learning through MProVE

The learning of Math can be a challenge for some students. Sometimes, it is not because of the subject but rather, their attitude and disposition towards learning it.

Focusing on Normal (Academic) Students

So long-standing is Dr Leong Yew Hoong's working relationship with Bukit View Secondary School that he is fondly known as Ah Leong by the teachers.

Eight years and counting of collaboration with the school has steered Yew Hoong's research towards exploring effective Math teaching for Normal (Academic) (NA) students.

NA students are often seen to be slower learners compared to the Express cohort, although their Math curricula are similar. "Certain ways of teaching the Express students may not be workable for the NA students," Yew Hoong notes. This motivated him to help them learn better.

Formulating a Supporting Framework

Bearing in mind the typical portrait of an NA student, Yew Hoong and his team, together with the teachers, conceptualized a framework of key learning objectives as part of the research project "Mathematical Progress and Value for Everyone (MProVE)".

These objectives serve as the team's reference points when addressing the NA students' difficulties in learning Math. From there, they produced a set of teaching resources.

We hope to see our students not shirking away, but engaging with the problem without necessarily having to solve it completely.

- Leong Yew Hoong, *Mathematics and Mathematics Education Academic Group*

The ultimate aim of the resources is to inculcate positive learning traits in NA students, by incorporating the following objectives:

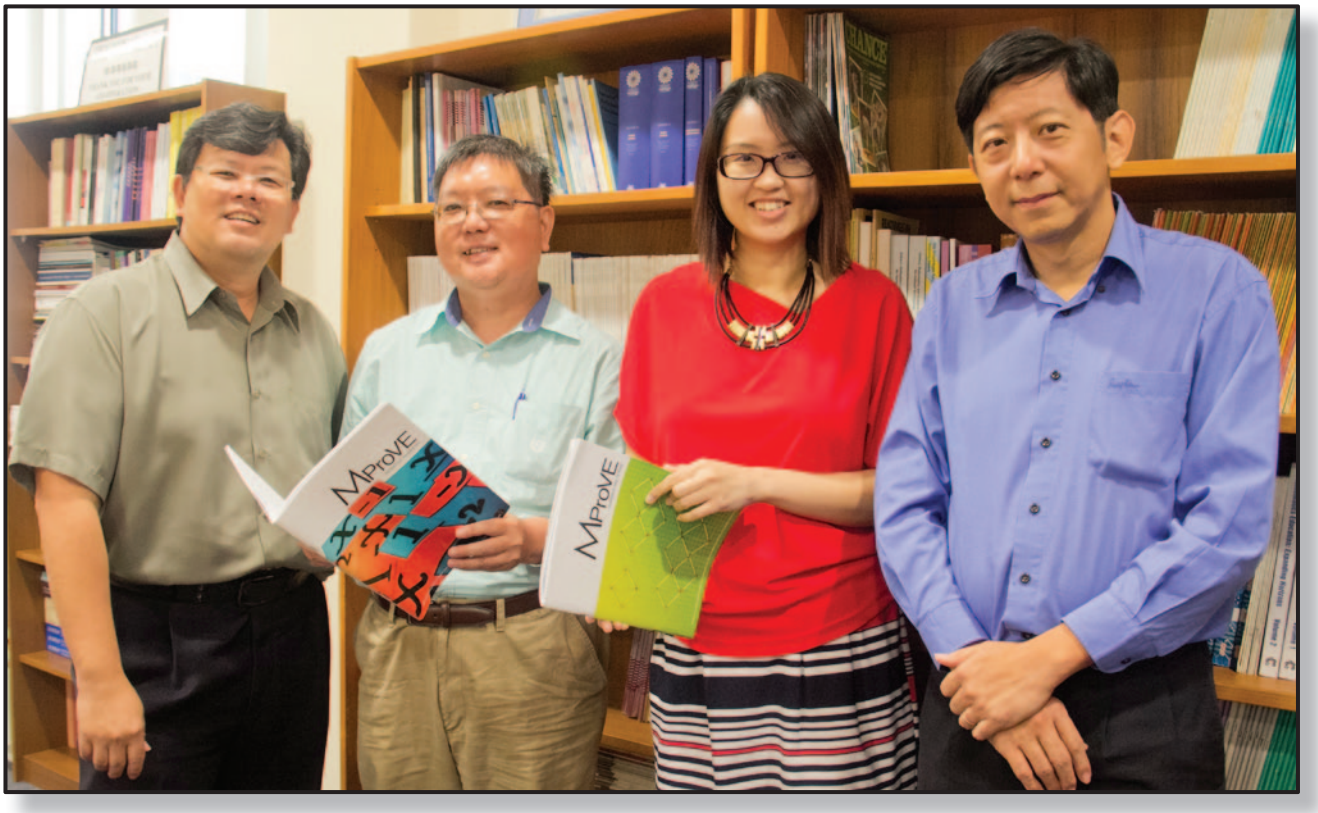
- It cannot be assumed that NA students are intrinsically motivated to learn Math. *Motivation* has to be brought in as a factor to develop positive learning attitudes.
- A *problem-solving disposition*, rather than a "wait-for-teacher-to-explain" attitude will help students tackle challenging problems. "We hope to see our students not shirking away, but engaging with the problem without necessarily having to solve it completely," says Yew Hoong.
- Good *study habits* should also be developed. Completing homework and taking down notes during class will help to reinforce and consolidate what they learn in class.
- Having *procedural fluency* means students will be familiar with basic mathematical procedures. Practice bolsters procedural fluency, which lets students tackle complex problems more easily.
- Last but not least, the *disciplinarity of Math* should also be instilled in the students. "Our project wants to retain disciplinarity. After the NA students have gone through our project, they should not just have good study habits, but also a sense of what Math is about, or what we call disciplinarity," elaborates Yew Hoong. "Math must be sensible. Students should be able to explain and understand a particular formula, and not just memorize it."

"Making Thinking Visible"

With these key learning objectives in mind, Yew Hoong's team worked with teachers to redesign Math units. Each "replacement unit" incorporates at least one of the objectives mentioned above, but procedural fluency and disciplinarity of Math—which are considered particularly important—are always present.

What is perhaps noteworthy is how the replacement units are especially rich in learning objectives, but they take the same amount of time as those units they replace. This is because the NIE researchers know teachers could not possibly conjure up extra hours in lesson time.

Through the process of redesigning the curriculum, teachers stumbled upon an interesting concept they dubbed "making thinking visible".



(From left) Tong Cherng Luen, Clement Lee, Karen Toh and Leong Yew Hoong are working together to help students adopt a more positive attitude towards Math.

When working on Math problems, teachers ask students to pen their reasons for solving a problem in a particular way in a column next to the questions. The teachers soon discovered that this was helpful not for just one topic, but a few topics across the board.

Yew Hoong explains, "That is the real intention of our project. It is not just about teaching this one topic better, but it is also hoped that by planning this number of replacement units, some of these skills are transferrable to other units which we did not plan."

In fact, such practices can be adapted for other subjects, and not just Math, says Yew Hoong.

Professional Development for Teachers

After a few rounds of tweaking, two Math units were finalized and published as print resources with CD-ROMs, while a few other units are in different stages of being refined.

More than just a project to enhance teaching and learning materials, Yew Hoong's work also serves to develop the teachers professionally.

Although the resources are free for dissemination among teachers, they "are not meant to be used without teacher development". He emphasized how there is much more to the successful implementation of such materials.

For example, the researchers video-recorded teachers as they implemented the replacement units in class. Taking into account the teachers' hectic schedules, the research team condensed 6 hours' worth of recording into 30-minute video summaries for the teachers' viewing.

A post-mortem discussion is conducted after each unit. Each teacher would point out interesting observations they gleaned from the video summaries, leading to further dialogue on improvements.

This kind of researcher-teacher partnership kills two birds with one stone: It develops teachers professionally, and creates an end product that is refined and proven to be effective.

Yew Hoong urges teachers who may be interested in this project to contact him, as he will be more than happy to work with them to help their Math learners.

Leong Yew Hoong is an Assistant Professor with the Mathematics and Mathematics Education Academic Group at NIE. His former experience in teaching Math at the junior-college level and subsequently secondary-school level has heavily influenced his research interest.

Classroom

Metacognition for Students and Teachers

Applying metacognitive skills in the Math classroom is important just so students understand what they are doing, instead of solving problems mechanically. Math teachers can also benefit from it when they prepare, conduct and reflect back on their lessons.

Have you ever stopped to consider why you always use a certain approach to solve a Math problem? Do you use it because it is the best, or because you are so used to it? Are you aware of the alternatives?

This is what metacognition is. In essence, it is "thinking about thinking" and being aware of the ability to control one's thinking process (Ministry of Education, 2012).

Developing metacognition has been an aim of Math education since the conceptualization of the Math curriculum framework in the 1990s. But even now, some teachers are still not confident that they can develop it in their students.

Recognizing that this is an important 21st century competency, Mdm Wong Lai Fong, a Lead Teacher in Anderson Secondary School, agreed to join Professor Berinderjeet Kaur from the Mathematics and Mathematics Education Academic Group at NIE in a project to enhance Math teachers' pedagogy for developing metacognition in their students.

> Verbalizing is very powerful. When students have to discuss or speak out, they have to *reflect* on what they are doing and *think* about what to say out loud.
>
> - Wong Lai Fong,
> *Anderson Secondary School*

Developing Metacognition

After drawing up the framework with Anderson Secondary, Prof Kaur spoke to their Cluster Superintendent to involve a few schools nearby. Then, selected Math teachers from these schools convened weekly to design tasks to be used in the classroom.

Lai Fong notes that it is not just students who benefit from developing their metacognition—teachers do, too!

Giving students the opportunity to solve non-routine and open-ended problems is the first part in helping them develop metacognitive awareness. For example, students have studied the topic of percentage in primary school—so teachers should ensure that students are not just answering questions mechanically and not understanding what they are doing.

"Instead of reinventing the wheel, we encouraged teachers to use existing questions and improve on them by changing performative tasks into knowledge-building ones."

"For example, when I ask, *'If I give you 20% of what I have in exchange for 10% for what you have, do you want to take that offer?'*, they have to think deeper about what percentage is and, hence, understand it," Lai Fong adds. "This makes them visualize their working backwards instead of doing it in the usual forward manner without thinking!"

Also important in developing metacognition is allowing the students to vocalize their thoughts. "Verbalizing is very powerful," Lai Fong says. "When students have to discuss or speak out, they have to *reflect* on what they are doing and *think* about what to say out loud."

Furthermore, by listening to how students support their answers, teachers will know if the students have understood the topic and can truly "make mathematical sense". This is a way of measuring if the students have indeed developed metacognition.

Practising "Teacher Noticing"

Students are not the only ones sharpening their metacognition in this project—teachers do, too!

After preparing the questions, the teachers in this project continue with meetings in their own schools, sharing the tasks with their own colleagues. But while putting them to practice, how do the teachers know if the pedagogy is actually working? "It's difficult for any teacher to teach, carry the task out and observe students' reactions and behaviour at the same time!" Lai Fong shares. "But we, too, have to have some form of metacognition and somehow reflect on our own lessons."

At Prof Kaur's suggestion, the teachers video-recorded their lessons and watched it afterwards. With so many things to look out for, Prof Kaur introduced the concept of "teacher noticing" to help teachers focus on specific and crucial elements of the lesson.

Lai Fong explains: "When we look back at our lessons, we observe it through four lenses: the teacher lens, student lens, task lens and climate lens. For example, with the teacher lens, we look at how the teacher asked the questions and carried out the tasks, while for the student lens, we look at their behaviour and reaction towards the said tasks."

"This is useful not only for the project, but also for any peer observations with my colleagues. We know now to focus on these various things and see how they can be improved on."

Encouraging Math Communication

Every classroom—not just a Math one—should have norms. What kinds of behaviour would the teacher want to promote in the students? Which are specific to Math?

Lai Fong points out that emphasizing on Math communication and literacy, which is being promoted in the new syllabus, is actually part of metacognition. It is important that students and teachers alike use the correct Math language.

"There are times when the teachers are teaching reducing of fractions, and instead of explaining that it's a process to divide both parts by a common term, they just say 'cancel' the numerator or 'strike' the denominator," Lai Fong shares.

"That's where students don't understand—whenever they see the same numbers, they will just strike them out even though it cannot be done! We want to eliminate that kind of wrong language and encourage the right Math classroom norm."

Reference

Ministry of Education. (2012, April). *O- & N(A)-Level Mathematics Teaching and Learning Syllabus.* Retrieved from http://www.moe.gov.sg/education/syllabuses/sciences/files/ordinary-and-normal-academic-level-maths-2013.pdf

Putting It All Together

Lai Fong notes that initially, her students were not used to the teachers asking so many questions. It was difficult to get them to talk more, and even harder to talk more mathematically.

"But the students got comfortable quickly! We want them to speak up and justify their solutions with mathematical reasons, and we are seeing more of that happening in class," Lai Fong enthuses.

She adds, "The teacher has to ask the right questions, and probe students to explain why they chose a particular operation instead of describing the operation. And when the students answer, the teacher has to listen and know what the students are really thinking."

While this might seem overwhelming for some teachers, Lai Fong advises teachers to start small and keep in mind how this paradigm shift will help students learn deeper.

"Time is always a factor when you're a teacher, but work within your means," she says. "You don't need a nice piece of worksheet—it can be just one question, done in one class. Most importantly, it helps to work with a group. When you have the support of your colleagues and management, you have shared resources and you can learn from one another."

Wong Lai Fong is a Lead Teacher (Mathematics) at Anderson Secondary School. She has been teaching for 19 years. She is involved in the project "Enhancing the Pedagogy of Mathematics Teachers to Facilitate the Development of 21st Century Competencies in their Classrooms (EPMT— 21st CC)".

Contributions

Learning, Doing and Using Math

In this article, the Principal of NUS High School of Mathematics & Science shares his views on Math education in Singapore and suggests future directions for both learning and teaching.

Contributed by Dr Hang Kim Hoo, NUS High School of Mathematics & Science

As both a Math educator and practising mathematician, I have held the simple slogan "Learn, Do and Use Mathematics" close to my heart. I had formulated the slogan while attempting to sort out and make sense of the views, exchanges, exhortations and insights among distinguished mathematicians and Math educators at the 2011 London Mathematics Education Summit, which was held at the Royal Institution.

This slogan has since provided me with a simple way to ensure that each time I am engaged in thinking about anything related to Math, I am able to mindfully keep a comprehensive and holistic view of the world of Math.

Indeed, at the heart of learning Math, it is always about learning correctly, accurately, comprehensively and competently, so that one will acquire the competencies and capacity to engage in lifelong and self-directed learning in Math.

This will then enable one to go on to do Math—to explore, build and develop new mathematical knowledge and tools (in the world of pure Math), and to use Math—to solve a whole range of real-world problems in tandem with knowledge from other disciplines (in the world of applied Math and mathematical modeling). Only then would one be able to appreciate the value, nature and beauty of Math.

In the world of Math, there is always a place for everyone. Engaging and leading our students in learning Math in schools should be a microcosm of the Math world described above. In fact, the current Singapore Mathematics Education Framework provides for that.

First introduced and adopted in 1990, and refined a number of times, it has enabled the development of the enacted Math curriculum in Singapore schools over the years and served the Singapore education system very well.

Dr Hang Kim Hoo notes that while Singapore's Math achievements have been commendable, more can be done to provide a total learning experience for all students.

The role of the Math teacher has evolved, shifting from one who presents content to one who enables students to learn how to learn.

- Dr Hang Kim Hoo *on why professional development for math teachers is important in moving forward*

Dr Hang Kim Hoo has more than 30 years of teaching experience and has held various appointments at the Ministry of Education. As Specialist Inspector, and Senior Curriculum Specialist for Mathematics at MOE, he was instrumental in steering the advancement of Math teaching and learning in Singapore.

It is noteworthy that as a nation, Singapore has done well in international Math benchmark studies (TIMSS and PISA) as well as at the peaks (Singapore has consistently been among the top 10 nations at the International Mathematical Olympiad since 2011).

Notwithstanding these achievements, there is still room for improvement in providing a total learning experience in Math for all our students.

Besides developing proficiency in mathematical conceptual understanding, skills and problem solving, there is room to discover how the various mathematical processes like reasoning, communication and connection making can be weaved more coherently to develop productive mathematical habits of mind.

At the same time, the introduction of the mathematical modelling process framework in 2010 by MOE should be leveraged upon as a platform to provide opportunities for students to use Math to unravel or even solve real-world problems in science, technology and social science contexts. Many of these problems can be made accessible to all students by varying the underlying conditions and assumptions, as well as making the contexts appropriate. The development of 21st century competencies can be woven in as well.

To take the next big step forward, continuous professional development for Math teachers has to focus on both content mastery and instructional strategies. The role of the Math teacher has evolved, shifting from one who presents content to one who enables students to learn how to learn, through the holistic development of mathematical knowledge, skills, competencies and productive habits of mind. In particular, content mastery among Math teachers will be critical as improving disciplinary mastery improves one's access to a wider repertoire of pedagogies.

If you would like to contribute an article to SingTeach*, please contact us at sgteach@nie.edu.sg.*

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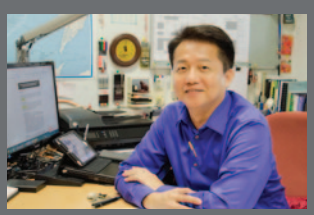

Metacognition in the Math Classroom

Motivating Students in Math

Flipping Math Teaching and Learning

The Discoveries of Reflective Practice